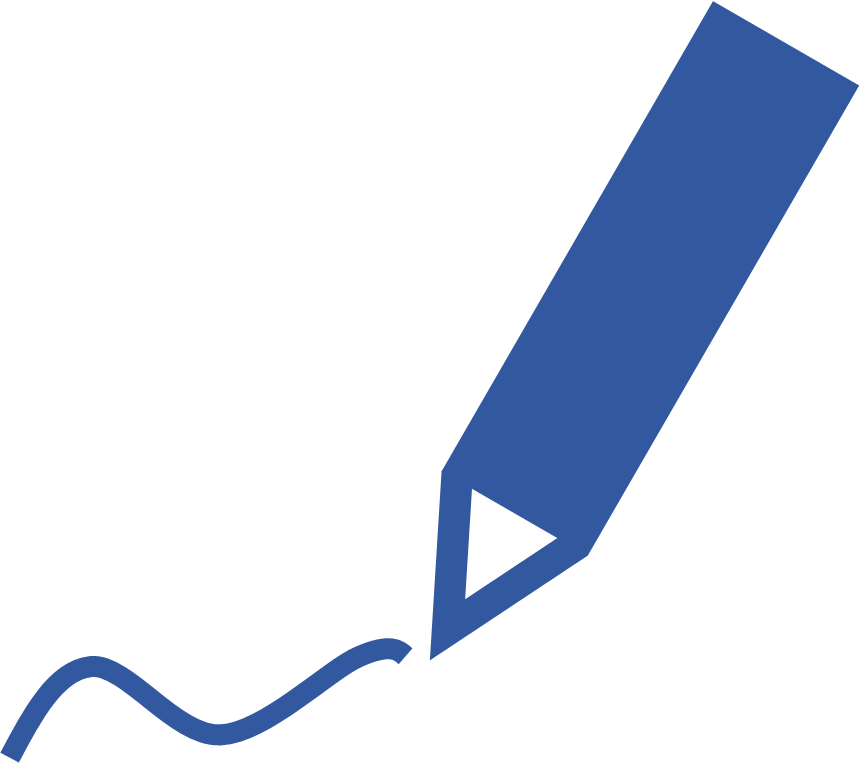
In the last section, the limit of the sum of infinity rectangles showed us the area under a curve. This is actually a method of evaluating the *definite integral*. This section will show what the definite integral is, where it’s used, how to evaluate it, and properties known to be true about the definite integral.

# The Definite Integral

The definite integral is the area under the curve.

The Definite Integral (object) – the combination of a definite[[1]](#footnote-1) range of infinitesimal data under the curve of .

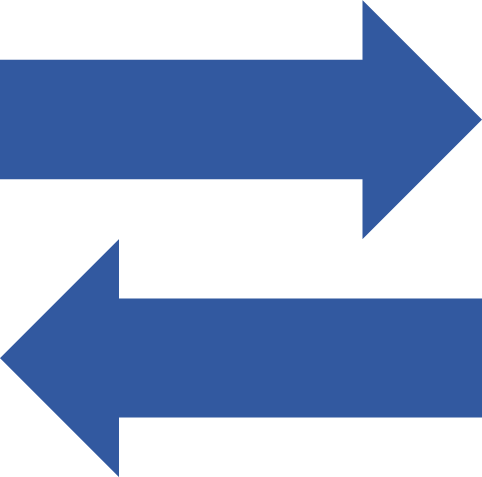
**Notation for integration**: .

## What Does the Definite Integral Mean?

To find the area under the curve from to , find the sum of infinity rectangles that make up the area (section 5.1). Those tiny slices each have the height of and the width of . The definite integral also tells us the area under the curve, and it is written like this:

Each part has a purpose in the definite integral. Consider:

|  |  |  |
| --- | --- | --- |
| Symbol | What it’s called | What it means |
| ∫ | Integration sign | This is the integral operation – the area. |
|  | Lower limit of integration | The integral starts at point . |
|  | Upper limit of integration | The integral ends at point . |
|  | Integrand | The height of each slice. |
|  | (no official name) | The width of the tiny slices that is across. |

**Connect the Dots**: . Replace Σ with ∫, ∆x with dx, and f(x\*) with f(x).

## Where Would You Use the Definite Integral?

Because integrals mean the area under the curve, you might use them to find antiderivatives (section 4.7) in a range. For example:

* A painter is painting the equation on a wall, where and are in feet and the -axis is 10 feet across the floor. He fills in all the area under the curve. How much paint will he use?
* You know a function that tells the speed of a rocket on the up-down axis. How far did the rocket move up or down over a certain duration of time?
* A 256-page book is written, starting at a rate of 1 page every day. Over the course of a week, the rate of writing gradually increases to one more page per day. How many days will it take for the book to be written?
* A factory line operates in direct proportion to the number of workers operating in it. If the number of workers is increased by 30% every hour, by what percent of average operations per hour will the factory have improved to over the next 4 hours?

## How Would You Do Evaluate the Definite Integral?

Area above the -axis is positive; area under the -axis is negative.

To find evaluate[[2]](#footnote-2) the definite integral :

1. Convert the integral to a summation of infinite, regular rectangles, using these formulas:
2. Apply algebra (section 1.1), summations rules (section 5.1), and limit laws (sections 1.3-6; 3.7).

**NOTE**: Area above the -axis is positive and adds to the integral’s area; area below the -axis is negative and subtracts from the integral’s area.

## A More Technical Defintiion of the Definite Integral

The Reimann sum is the sum of infinity rectangles. Their heights are , where is any value in subinterval .

If a function exists, then in the interval for any integer , there are number of **subintervals**. All them together are called a **partition** of .

The th subinterval has the width .

**Samples points** are chosen, one for each subinterval, written . They can be anywhere in the subinterval; on the far-left, far-right, right in the middle, or somewhere in-between.

The **midpoint rule** lets the sample point be in the middle of its subinterval. .

The **Reimann sum** for a partition is the sum of the areas of infinity rectangles defined from the partition described above. (This means that if a rectangle goes *under* the -axis, it has a *negative* area, and would *subtract*, not add, its area from the sum)

Since the width of each subinterval () can be different, to find the *definite* integral, limit the maximum of to . The **definite integral** then is this:

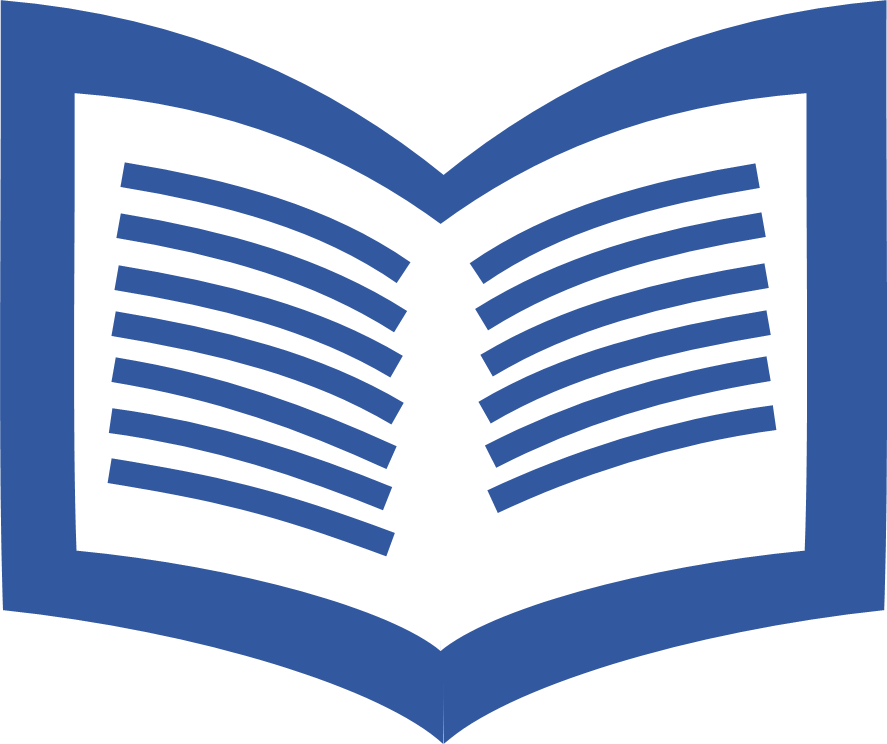
## Properties of the Definite Integral

Like many mathematical objects, integrals can be converted between forms using rules. Here are some:

|  |  |  |
| --- | --- | --- |
| If it looks like this… | …then it can be this. | Here’s why |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

As well as properties that convert from one form to another, integrals have properties that tell how they *compare* with other expressions.

|  |  |  |
| --- | --- | --- |
| If this is true… | …then this is true. | Here’s why |
| If for , | then |  |
| If for , | then |  |
| If for , | then |  |

**Look in the Book**: for proof that these rules are true, see pages 276-8 in the textbook.

# What Did You Learn?

* What is the definite integral? How do you evaluate it? Where would you use it?
* What are some properties of the definite integral? What do they do? Why are they true?
* What is another way to evaluate the definite integral? Why is it true? How does it do this?

1. **Definite**: clearly defined; explicitly precise. [↑](#footnote-ref-1)
2. **Evaluate**: find the value of. [↑](#footnote-ref-2)